Dominating Connectivity and Reliability of Heterogeneous Sensor Networks

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Two Issues Effecting to the Reliability of Wireless Sensor Networks

1. Sensors that run on batteries have limited energy reserves, e.g., may run on two AA batteries.

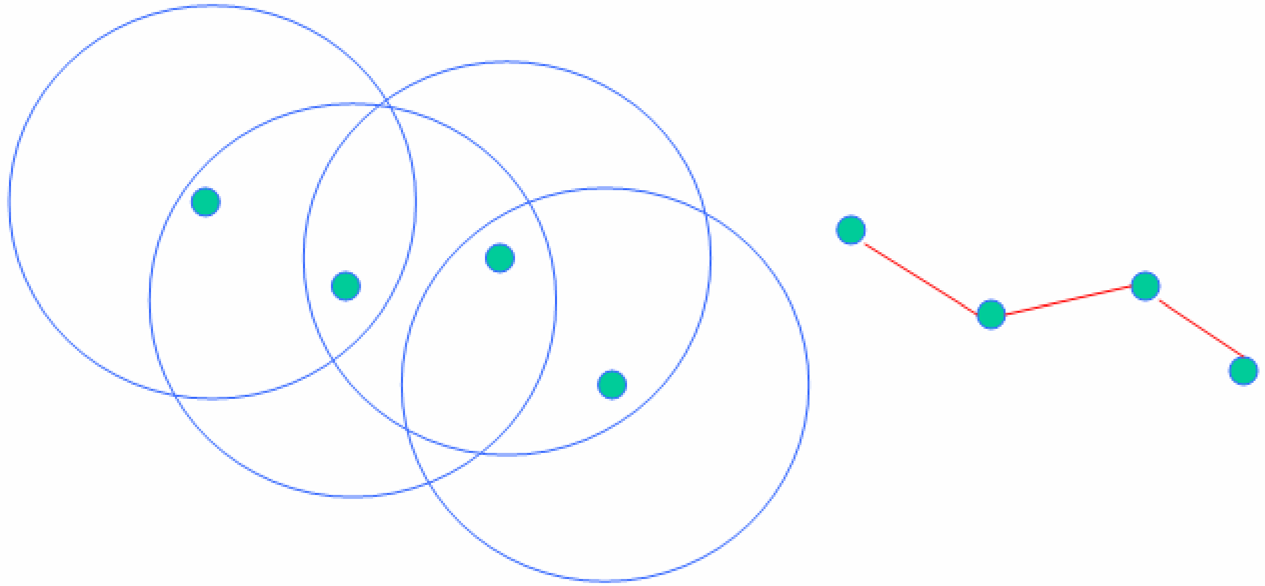
2. Enemy agents may sabotage network.

Energy conservation

In many WSN applications, using and replacing batteries is impractical. Often the sensor nodes are located in unrecoverable locations. Furthermore, the labor and costs associated with changing hundreds, if not thousands, of batteries outweighs the ROI (Return on Investment) that the sensor network could deliver.

Thus it is important to conserve energy when routing.

Multi-hop routing



Multi-hop routing saves power

Power required for sensor *A* to reach sensor *B* is a quadratic function of the distance between *A* and *B*. Thus, a sequence of smaller transmissions to intermediate nodes requires less total power than a direct communication.

Fault-tolerance & security

- **Designing and analyzing fault-tolerant networks**
  - networks with the ability to survive (remain connected) even if there are sensors/node failures throughout the network.
- **Routing schemes for gather, broadcast and point-to-point communication that involve routing along multiple node-disjoint paths.**
  - Such schemes that can detect and circumvent enemy agent that is trying to disrupt the routing scheme.

Homogeneous vs. heterogeneous sensor networks?

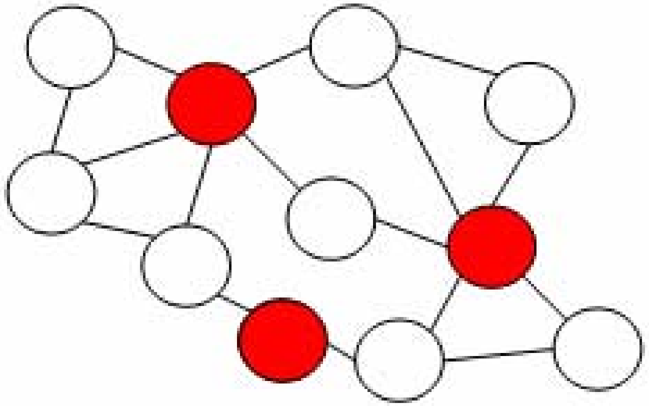
- **Homogeneous Sensor Networks:**
  - All nodes transmit with same power
  - Identical transmission radius – inefficient
- **Heterogeneous Sensor Networks:**
  - Each node has its own transmission radius
  - How to assign radius?

Traditional *k*-connectivity

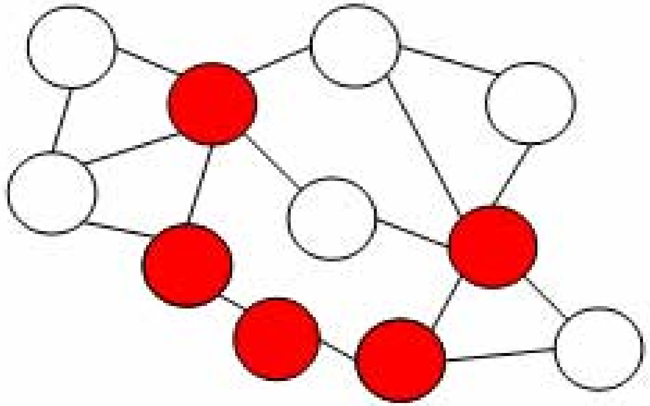
- A network is *k*-connected if it remains connected even after the deletion of any *k* nodes.

- Equivalently, it is *k*-connected if there are *k* pair-wise disjoint paths joining any pair of nodes.

Dominating Set



Connected Dominating Set



New concept

Dominating k-connectivity

We will say that a network is *dominating k-connected* if, after the simultaneous failure of at most $k - 1$ nodes in the neighborhood of any surviving node, the surviving nodes form a connected dominating set.

Dominating *k*-connectivity vs. traditional *k*-connectivity

Dominating *k*-connectivity is a much stronger connectivity property than *k*-connectivity.

However, we show via theoretical results and empirical testing that to achieve dominating *k*-connectivity in a heterogeneous sensor network requires only a small increase in the radii of transmission over that required for *k*-connectivity.

Dominating *k*-connected to a sink node

We say that a network is *dominating k-connected to a sink node s*, if the network is able to resist simultaneous node failures throughout the network and still remain connected to the in-neighborhood of *s*, provided that at most $k - 1$ nodes have failed in the out-neighborhood of any surviving node.

Thus, a network is dominating *k*-connected iff it is dominating *k*-connected to every node.

K-bounded subset

A *k-bounded subset* $G[U]$ is one where every node in U is adjacent to at most $k - 1$ nodes not in U.

Thus, a digraph is dominating k-connected to s iff every k-bounded set U induces a subgraph $G[U]$ that contains a directed path from each node of u in U to the in-neighborhood of s.

Characterization of dominating *k*-connected networks

Theorem 1: Let $G = (V, E)$ be a digraph, and suppose s is any node of G. Then, G is dominating k-connected to s if and only if there exists a linear ordering of the vertices, such that each node v different from s, either contains s in its out-neighborhood, or contains at least k nodes in its out-neighborhood that are smaller than v with respect to the linear order.

Proof of Theorem 1 (if)

Since there are at most $k - 1$ neighbors of u that do not lie in U, there is at least one node $u_1 < u$. Applying the same argument with u_1, there exists a node u_2 in U such that $u_2 < u_1$. Continuing in this way, we obtain a sequence

$$u > u_1 > \ldots > u_j$$

Since the nodes in the sequence are successively smaller in the linear order and eventually a node in the in-neighborhood of s must be reached.

K-smaller property

A subset *A* of *V* has the *k-smaller property* if there exists a labeling *L* of the nodes in *A*, such that for each node *a* in *A* that is not in the in-neighborhood of *s*, there exists at least *k* nodes in the out-neighborhood of a that belong to *A* and have a strictly smaller label than *a*.

Proof of Theorem 1 (only if)

Consider a set A of maximum size having the k-smaller property, and consider the set $U = V \backslash A$.

We will show that U is *empty*, so that $A = V$. Assume to the contrary that U is not empty.

Suppose u in U is joined to at least k nodes not in U. Then, by assigning u a label greater than any label on the nodes of A, the set $A + u$ would have the k-smaller property contradicting the maximality of A. Thus, U is k-bounded.

But, since $A = V \backslash U$ contains the in-neighborhood of s, $G[U]$ is not connected to the in-neighborhood of s, contradicting the fact that G is dominating k-connected to s. Hence, $A = V$.

Dominating Connectivity

Dominating connectivity to a sink node s is the maximum *k* for which *D* is dominating *k*-connected.

Dominating connectivity of D is the maximum *k* for which *D* is dominating *k*-connected.

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Dominating Connectivity Algorithm

procedure *DominatingConnectivitytoSink*
Input: Graph $G = (V, E)$ having n nodes
Output: Dominating connectivity κ_s to a sink node s in G

$A = N_{in}(s) \cup \{s\}$
$\kappa_s = n$
while $A \neq V$ **do**
  for each node $u \in V \backslash A$ **do**
    $\epsilon(u)$ = number of edges ua such that $a \in A$
  endfor
  b = a node $u \in V \backslash A$ that maximizes $\epsilon(u)$
  $A = A \cup \{b\}$
  $\kappa_s = min\{\kappa_s, \epsilon(b)\}$
endwhile
end *DominatingConnectivitytoSink*

Algorithm for Optimal Radii Assignment

procedure *OptimalDCRadiustoSink*

Input: A distance function d mapping $V \times V$ to the real numbers, representing distances between sensors, where one sensor s is designated as the sink, and a positive integer k

Output: Optimal transmission radius assignment r, such that the associated network is dominating k-connected to s

$A = \{s\}$

$r(s) = 0$

while $A \neq V$ **do**

  for each node $v \in V \backslash A$ **do**

    $M(u) = min\{d(u,s), d_k(u,A)\}$

  endfor

  b = a node $u \in V \backslash A$ that minimizes $M(u)$

  $r(b) = M(b)$

  $A = A \cup \{b\}$

endwhile

endfor

end *OptimalDCRadiustoSink*

Theorem 2: Let $G = (V, E)$ be a digraph that is strongly connected. Then the k^{th} power G^k of G is dominating k-connected.

Theorem 4: The k-expansion the network M resulting from placing sensors at the points of a square two-dimensional mesh in the plane, i.e., at the points $\{(i, j)|i, j \in \{0, 1, \ldots, q\}\}$, satisfies

$$\epsilon_k(M) \leq \sqrt{2}\lfloor\sqrt{k} + .5\rfloor.$$

K	Max Expn Dom	Avg Expn Dom	Avg Expn Degree
1	1	1	0.918163
2	1.50768	1.19746	1.115463
3	1.66048	1.35657	1.270327
4	2.12019	1.50813	1.436498
5	2.21503	1.59920	1.528536
6	2.24924	1.72023	1.653984
7	2.32857	1.81776	1.737909
8	2.54519	1.89741	1.824917
9	2.67664	2.01218	1.922366
10	2.68461	2.09472	2.018564
20	3.67839	2.87293	2.825125
30	4.30078	3.47126	3.424622
40	4.99854	4.00779	3.952914
50	5.68990	4.45570	4.388446
60	6.28236	4.86740	4.794621
70	6.78365	5.29621	5.221444
80	7.15769	5.62992	5.547579
90	7.68274	5.99429	5.906015
100	8.10678	6.31972	6.223447

TABLE I

EXPANSION VALUE STATISTICS FOR ACHIEVING DOMINATING k-CONNECTEDNESS, WHERE THE SECOND AND THIRD COLUMNS FROM THE LEFT CORRESPOND TO THE MAXIMUM AND AVERAGE EXPANSION, RESPECTIVELY, NEEDED TO INDUCE A DOMINATING k-CONNECTED NETWORK AND THE FOURTH COLUMN CORRESPONDS TO THE AVERAGE EXPANSION NEED TO INDUCE A NETWORK OF MINIMUM DEGREE k.