Hardness and Approximation of gathering in radio networks

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Project MASCOTTE - INRIA/I3S/CNRS
European project AEOLUS
CONICYT/INRIA Sophia-Antipolis
CRC CORSO funded by France Télécom R & D, on *Design of Telecommunication networks, fault tolerance and radio networks.*

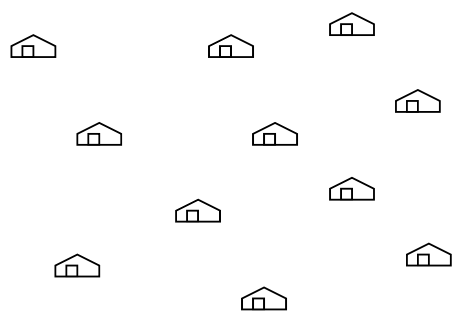
Contents

- Motivation.
- The Model & The Problem.
- Results.
- Proof drafts.
- Conclusions.

MOTIVATION: HOW TO TAKE INTERNET TO THE VILLAGES...

- ...without wires?
- **Idea:** Provide the houses with wireless devices and put a central antenna that communicates them with the world.
- Similar to mobile phones, but without direct connection to the central antenna: multi-hops.

TAKING INTERNET TO VILLAGES



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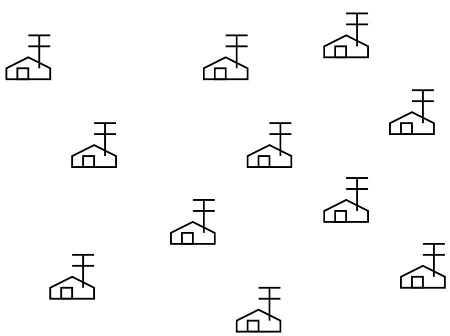

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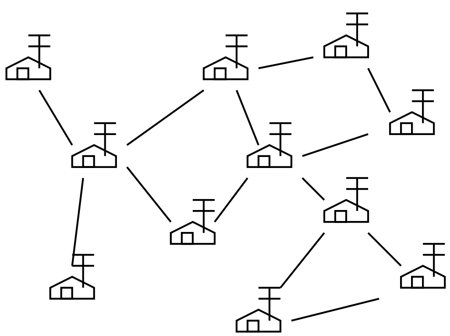

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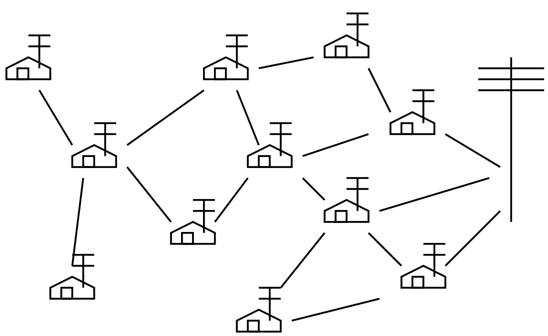
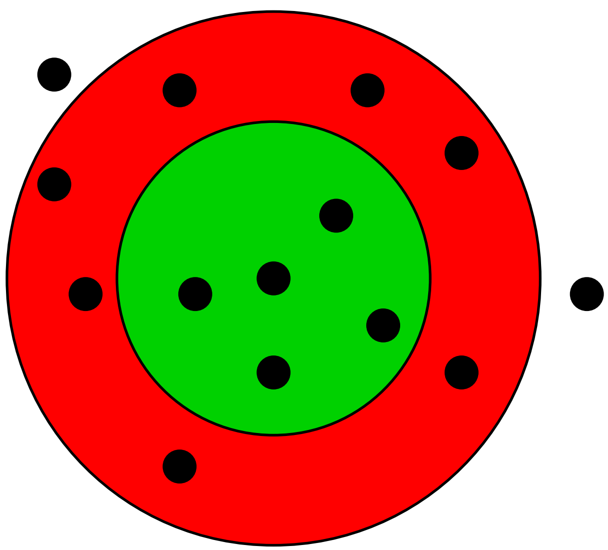
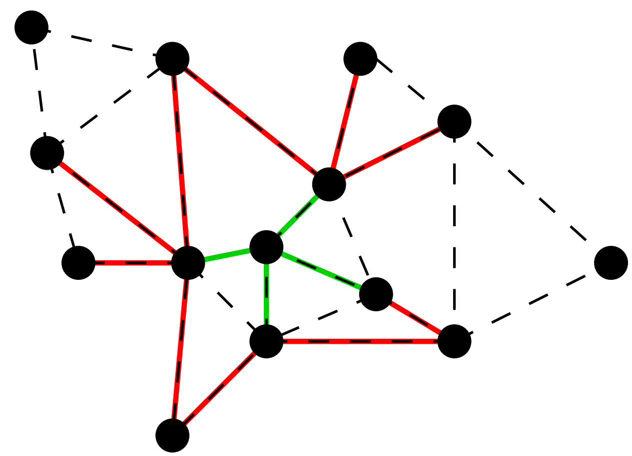


THE RULES OF THE GAME

- Rule #0: Universal clock.
- Rule #1: In a given round, a device either receives or transmits.
- Rule #2: A device can transmit only one piece of information per round.
- Rule #3: Devices transmit up to a distance d_T.
- Rule #4: If u transmits and $d(u, v) \leq d_I$, then v cannot receive from a third node $w \neq u$ (interference is call-to-call).

THE RULES IN A PICTURE



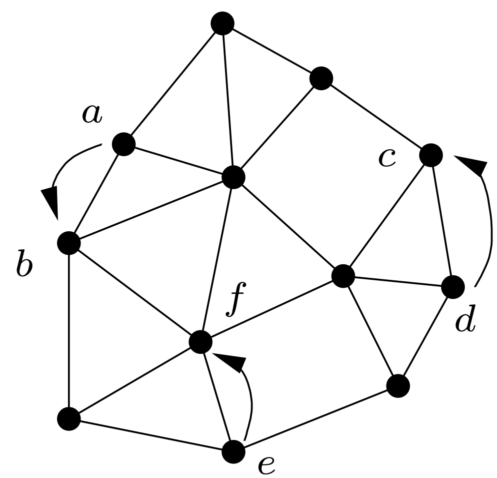


$$d_I = 2, d_T = 1$$

THE MODEL

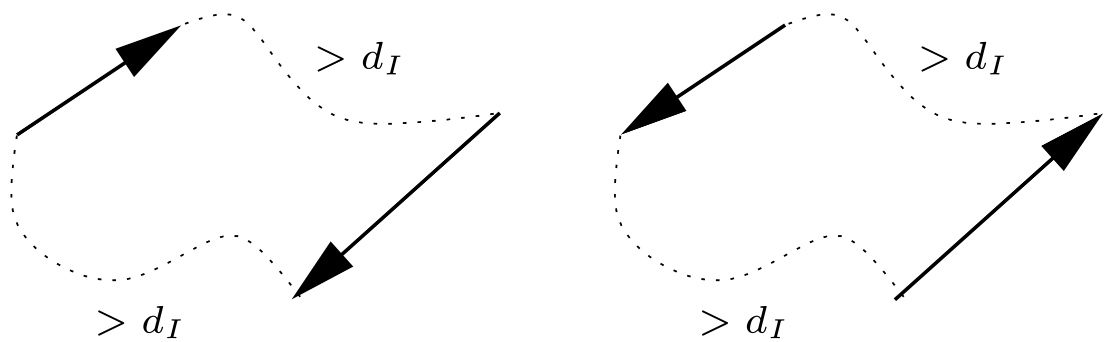
- A graph $G = (V, E)$, a sink $t \in V$ and two numbers d_I, d_T. For any $u \in V$, a number of messages $w(u)$.
- We construct:
  - the transmission digraph $G_T = G^{d_T} = (V_T = V, E_T = \{(u, v) : d(u, v) \leq d_T\})$ ($e \in E_T$ is said a *call*), and
  - the interference graph $G_I = (V_I = E_T, E_I = \{[e, f] : e$ and f interfere$\}$.
- A valid *round* is an independent set of G_I (i.e. a set of calls that are non-interfering).
- GOAL: Find a sequence of rounds that routes all the messages up to the sink in minimum time (number of rounds). We call $g(G, t)$ such a minimum.

THE MODEL — A FIGURE



If $d_I = d_T = 1$, a round may consist of the three calls $(a, b), (d, c), (e, f)$, while if $d_I = 2$, then (e, f) interferes with (a, b) and (d, c) (if the three calls take place, only c receives!). A round may consist of a single call or the calls (a, b) and (d, c).

FOR THE CURIOUS: WHAT ABOUT *personalized broadcasting*?



Related work

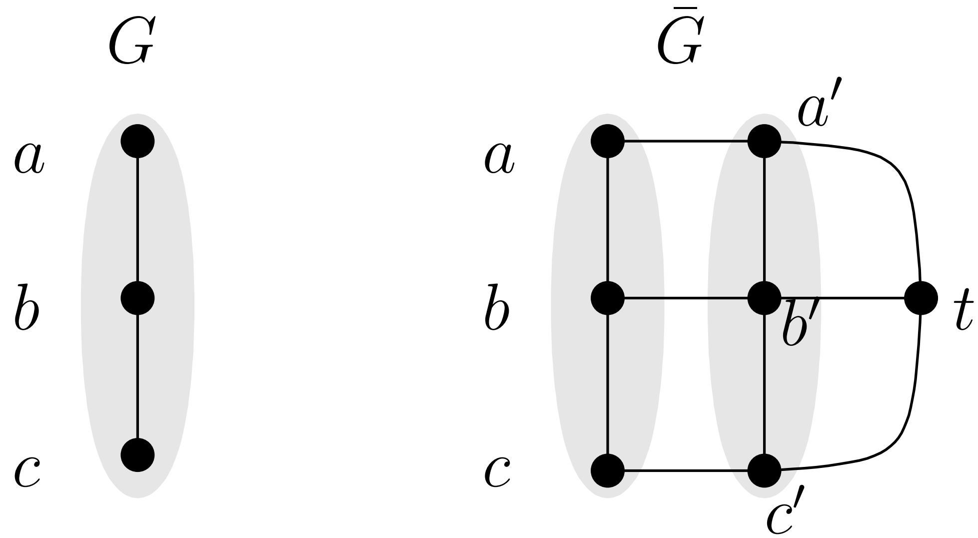
- J.-C. Bermond and J. Peters (Algotel 2005): Optimal protocols for gathering into the center of the **2D-Grid**, when $d_T = 1$.
- J.-C. Bermond, R. Correa, J. Yu (CIACC'06): Very good (sometimes optimal) protocols in the **path**.
- —: For a related model, general flow demands $f(u, v)$. NP-Hardness and Approximation. Optimal strategies for the path and trees. PTAS for Euclidean graphs and subsets of the grid.

OUR RESULTS

- Hardness: In general, the problem is NP-COMPLETE. Actually, if $d_I > d_T$ it does not admit a FPTAS. (Approximate with quality ϵ requires time that is exponential on $1/\epsilon$.)
- Approximation: General algorithm + General lower bound $\Rightarrow$ 4-Approximation.
- *Good* protocols for specific cases.

HARDNESS — NO FPTAS WHEN $d_I = 2, d_T = 1$

- Given a graph G that we want to color, we construct an auxiliary graph $\bar{G}$ as follows



- Key fact: The calls $(x, x'), (y, y')$ are compatible if and only if x, y are independent in G.
- If $(\forall x)\, w(x) = w(x') = 1$, it follows that $g(\bar{G}) = \chi(G) + 2n$.

Approximation — General lower bound

- There exists a bottleneck close to the sink, in which at most one message can be received per round. Therefore, each message must travel this section *alone*.(*)
- For instance if we focus on balls of radius r_c centered at the sink, (*) is true for $r_c \leq \lfloor \frac{d_I - d_T}{2} \rfloor$.
- We get that $g(G,t) \geq \sum_{u \in \Gamma^{r_c}} \frac{w(u)d(u,t)}{d_T} + \frac{1+r_c}{d_T} \sum_{u \notin \Gamma^{r_c}} w(u)$
- **Very roughly** (not true): $g(G,t) \geq \frac{d_I - d_T + 1}{d_T} W(G)$, $W(G)$ being the total number of messages.

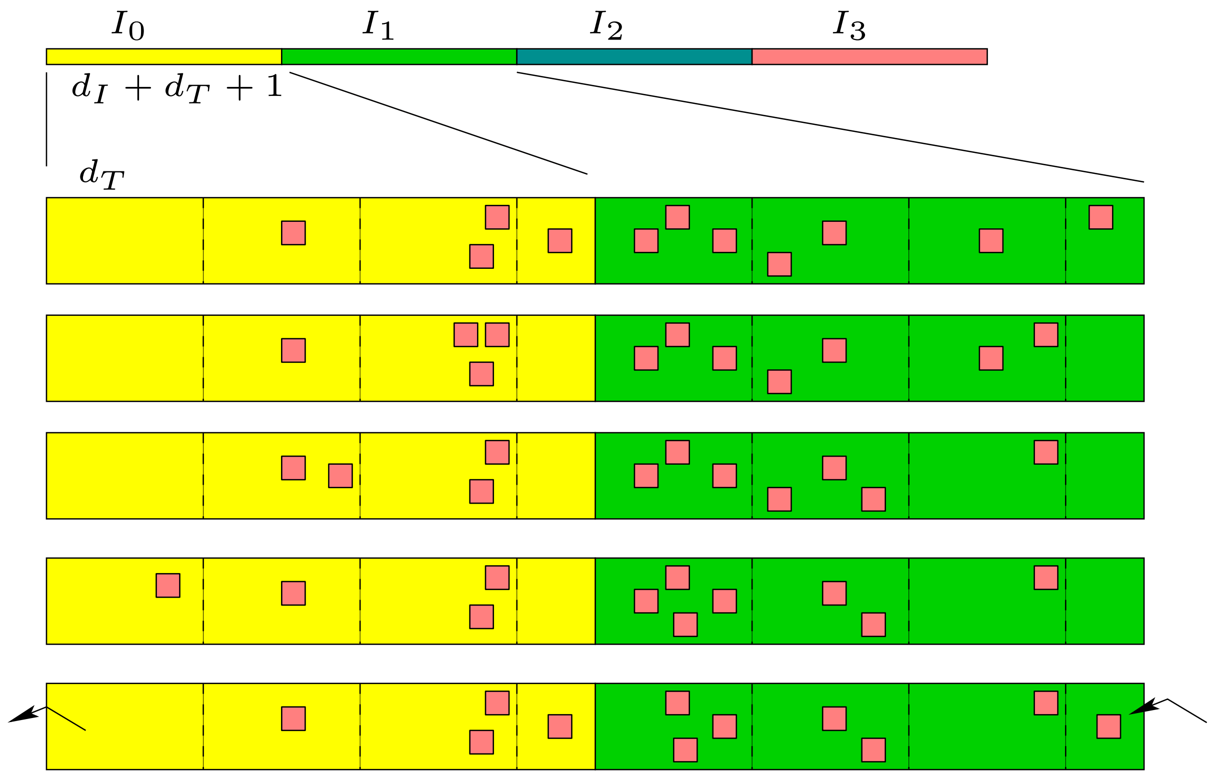
APPROXIMATION — ALGORITHM FOR THE PATH

- We take a path $P_n = \{0, \dots, n-1\}$, vertex i connected with vertex $i+1$, sink $t = 0$.
- We divide P_n into intervals of length $d_I + d_T + 1$, so

$$
\begin{aligned}
I_0 &= \{1, 2, 3, \dots, d_I + d_T + 1\}, \\
I_1 &= \{d_I + d_T + 1, d_I + d_T + 2, \dots, 2(d_I + d_T + 1)\} \\
I_k &= \{k(d_I + d_T + 1) + p\}_{p=1}^{d_I + d_T + 1}.
\end{aligned}
$$

- Main idea: pipeline. Every interval transmits one message to the previous one in about $(d_I + d_T + 1)/d_T$ rounds.

ALGORITHM FOR THE PATH

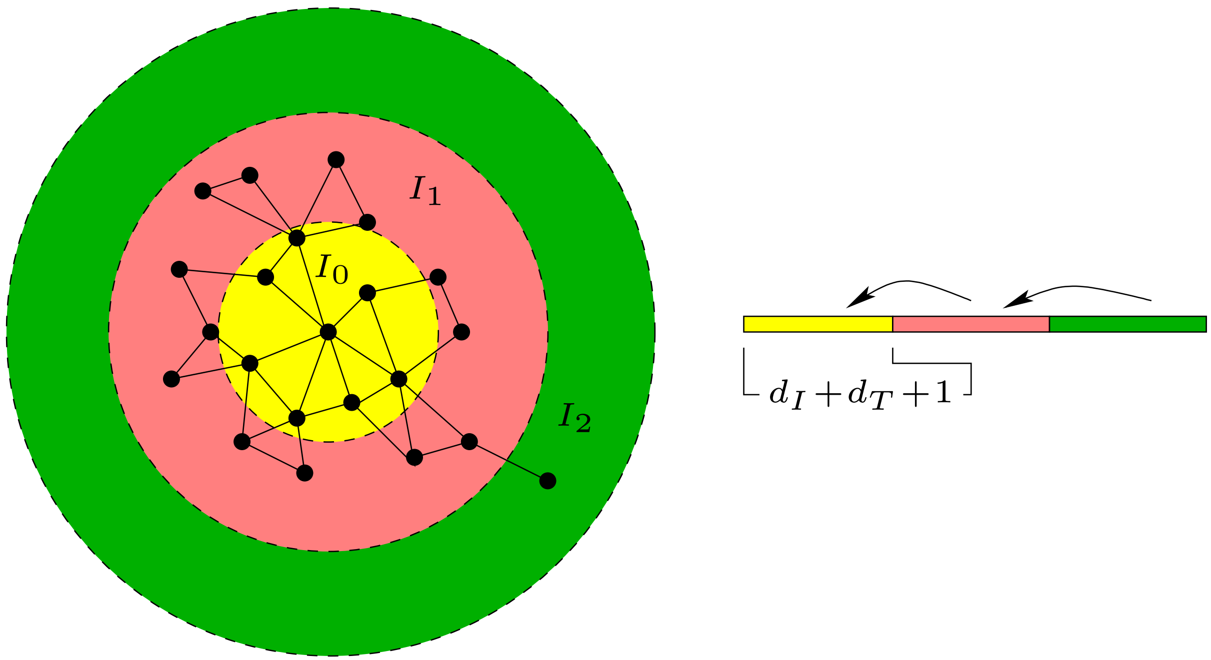


It follows that $g(P_n, t = 0) \leq \frac{d_I + d_T + 1}{d_T} W(G)$. (Again, roughly, not true.)

ALGORITHM FOR GENERAL GRAPHS

Exactly the same idea can be applied in a general graph, if we consider interval k defined as the vertices x such that

$$k(d_I + d_T + 1) < d(x, t) \leq (k+1)(d_I + d_T + 1)$$



It follows the same bound that for the path!

ANALYSIS OF THE ALGORITHM

- The rough analysis:

$$\text{approx. ratio of alg.} \leq \frac{\frac{d_I+d_T+1}{d_T}W(G)}{\frac{d_I-d_T+1}{d_T}W(G)} \leq 4.$$

- The actual proof has several cases, considers separately the weight of Γ^{r_c}, takes care about empty intervals, etc, etc.
- The bound is 4 independent of d_I, d_T, but for many cases the value is smaller. In particular, the ratio goes to 2 as $d_T/d_I \to 0$.

BETTER RESULTS FOR SPECIFIC CASES

- We say that a protocol A is *nearly optimal* if the gap with respect to the optimum value does not increase with the size of the network. ($A(G,t) - g(G,t) \leq C = C(d_I, d_T)$.)
- The path. The algorithm above described is *nearly optimal*.
- Balanced stars, in the uniform case:
  - Nearly optimal protocols by, either improving the lower bound (2 branches) or using a better algorithm (parallelizing).
  - Attain the general lower bound.
  - Show that the analysis of the algorithm is tight (4-approximation).
- What applies to balanced stars is true for the 2D-grid.

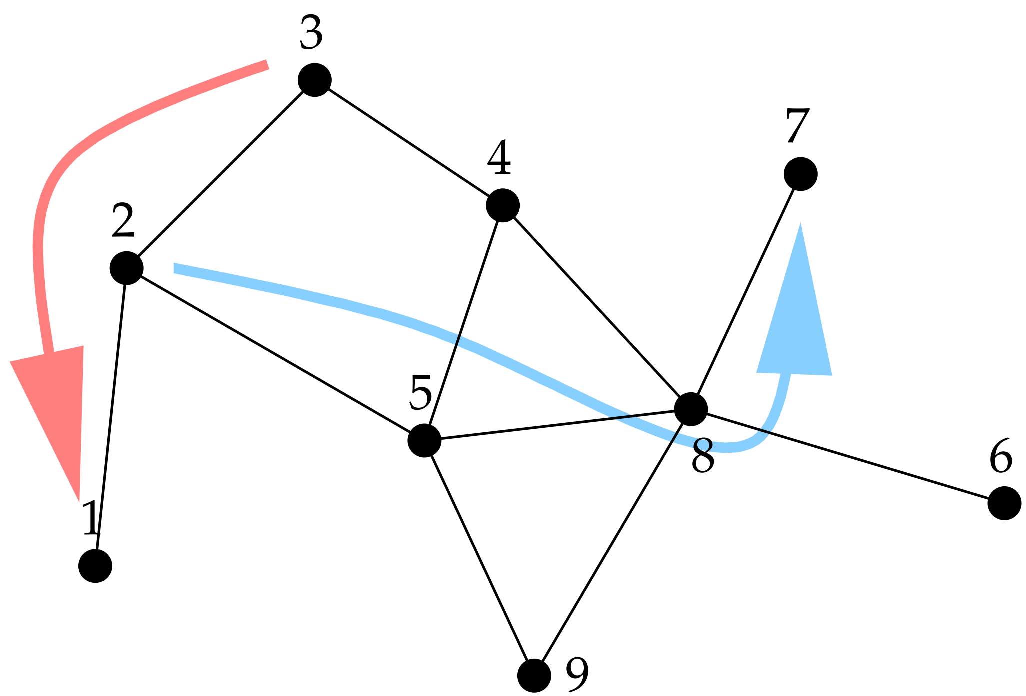
THANK YOU !

MERCI !

GRACIAS !

GRAZIE !

Bandwidth allocation in classical networks

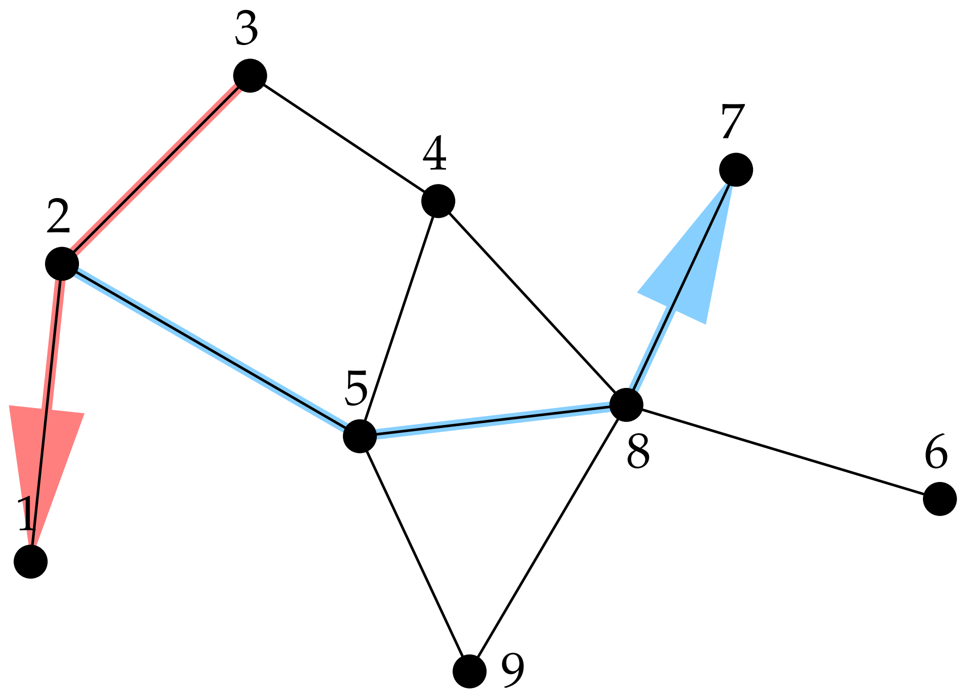


Traffic demands:
- 1 wants 3 units from 3
- 2 sends 2 units to 7

...

A function $f(u, v)$ spacifies the desired traffic from u to v

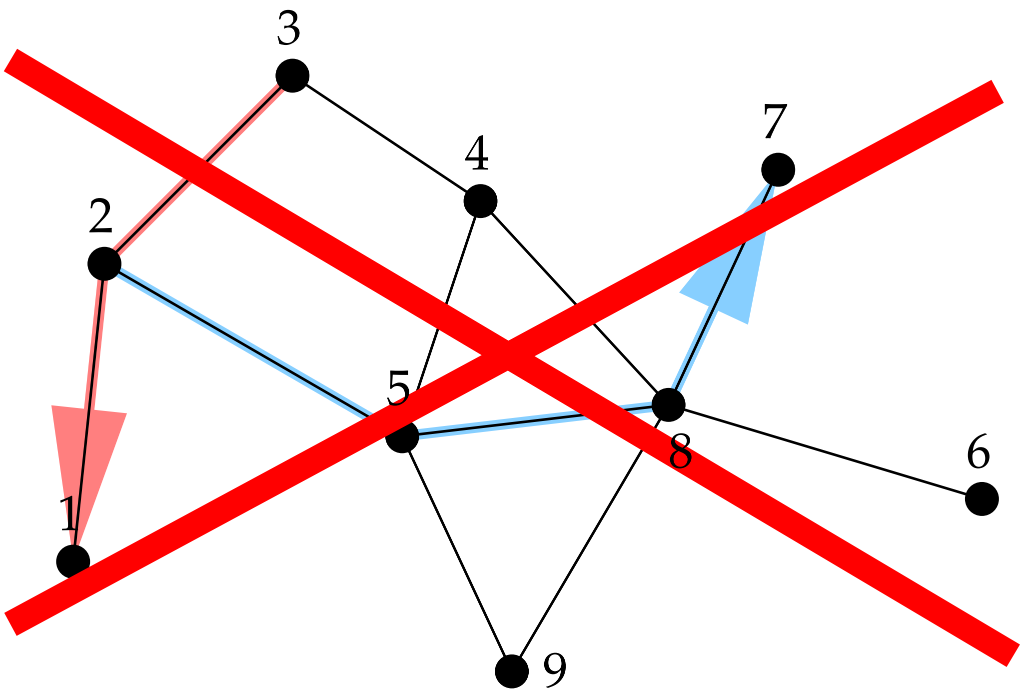
Bandwidth allocation in classical networks



Traffic demands:
- 1 wants 3 units from 6
- 4 sends 6 units to 7
...

A function $f(u, v)$ spacifies the desired traffic from u to v

BANDWIDTH ALLOCATION IN RADIO NETWORKS



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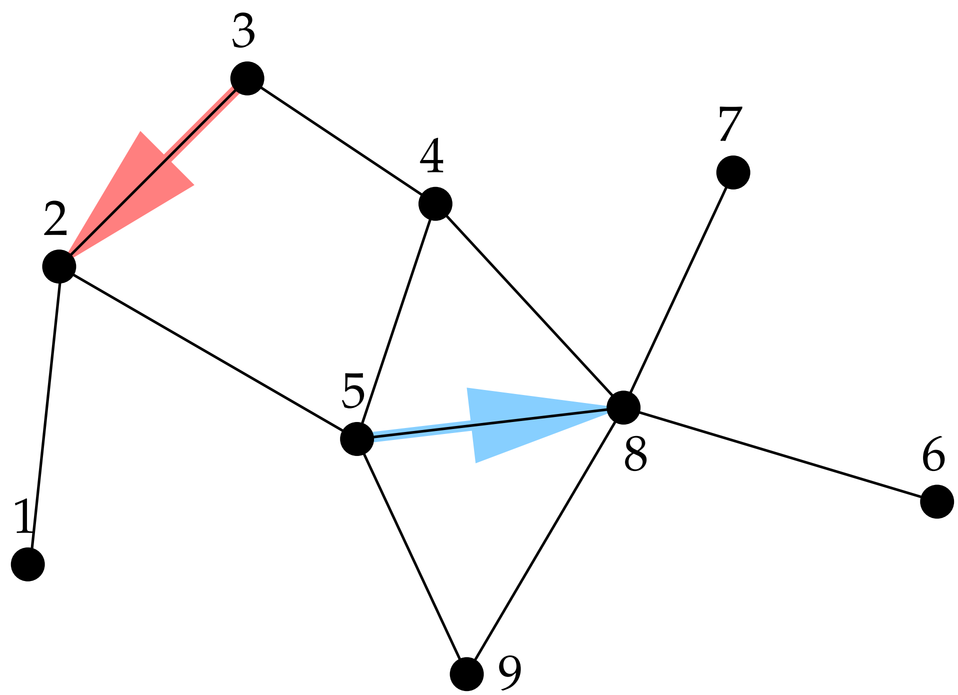

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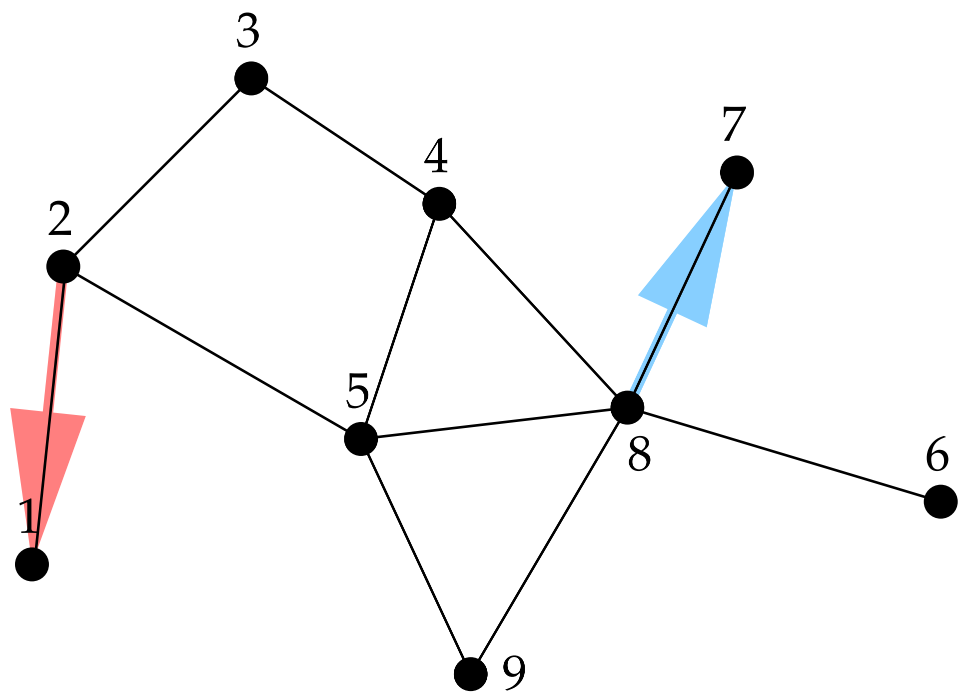

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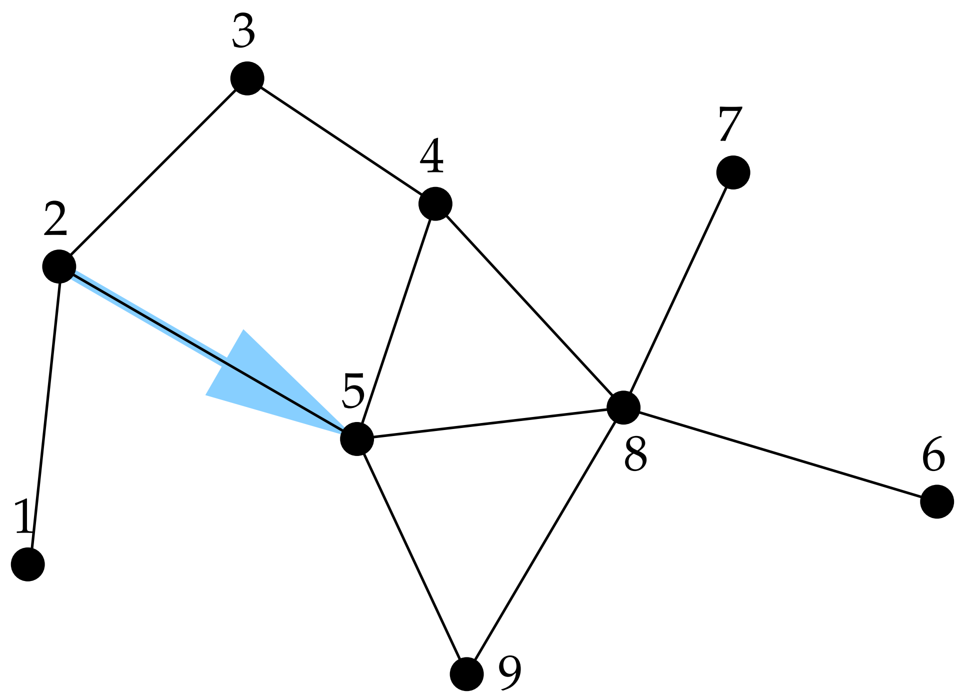

Bandwidth allocation in radio networks