Complexity of Scheduling in Multi-hop Wireless Networks

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Multi-hop Wireless Networks

- Characteristics:
  - Mutual interference between scheduled links
    - variable link capacities
  - Capacity region of the network depends on routing, link activation and power allocation
  - Coupling across multiple layers of the protocol stack

Efficient operation of such networks requires a cross-layer design approach!

Cross Layer Design

- Optimize jointly across multiple layers of the protocol stack
  - Joint rate control, routing, and scheduling
- Cross layer problems exhibit a nice decoupling property (see, for example, [Lin & Shroff, CDC 2004])
- The scheduling component is the central and most difficult element of all cross layer optimization frameworks

Cross Layer Scheduling Problem

- The cross layer scheduling problem is

$$r = \underset{r \in u(P), P \in \Pi}{\arg\max} \sum_{l=1}^{L} q_l r_l$$

Data rate of link l

Congestion price at link l

Rate region

- No simple characterization possible even for u(P) in terms of P!
- What to do?

Imperfect Scheduling Schemes

- Scheduling schemes that are within some factor of the optimal
- S_β-scheduling schemes: Schemes that are guaranteed to choose a rate vector s that satisfies:

$$\sum_{l=1}^{L} q_l s_l \geq \beta \max_{r \in u(P), P \in \Pi} \sum_{l=1}^{L} q_l r_l$$

- Using a S_β-scheduling scheme in the cross layer framework one can achieve a capacity region of β times the optimal [Lin and Shroff, INFOCOM 2005]

Wireless Network Model

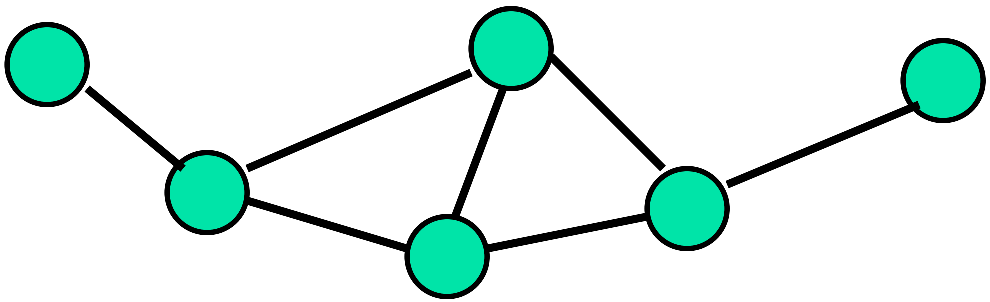
- Fixed transmission powers
  - Each node transmits at some fixed power level when scheduled
  - The power level can differ across nodes
- This reduces the cross layer scheduling problem to a combinatorial optimization problem
  - Still quite difficult to solve – requires centralized control!

Interference Models

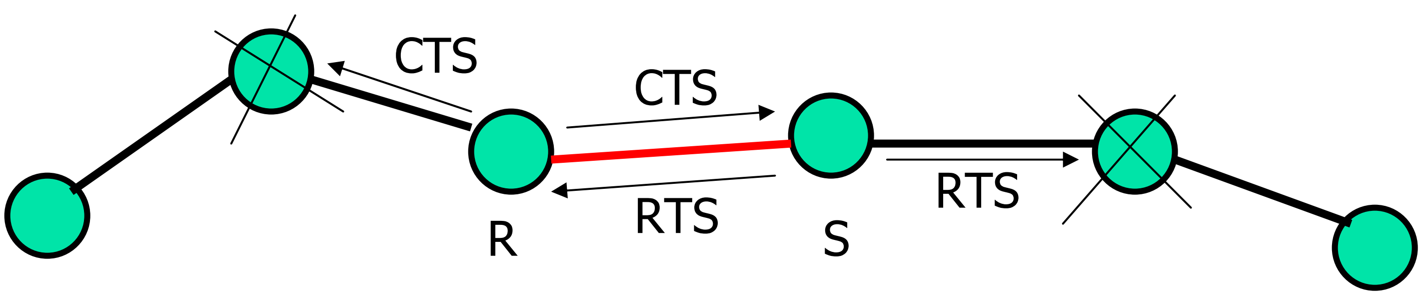
- K-hop interference models
  - Limits interference to K hops
  - Links within K hops of each other cannot be scheduled to transmit at the same time
- Consider only bidirectional links
  - Required by most current network and transport layer protocols

K-hop Interference Models

- K=1: Node Exclusive Interference Model



- K=2: Models the RTS/CTS based communication scheme of IEEE 802.11



Terminology

- Model the wireless network as an undirected graph $G=(V,E)$
  - V is the set of nodes
  - E is the set of edges/links
- A subset of edges $M \subseteq E$ is called a *K-Valid Matching* if and only if all edges in M are at least K hops apart from each other
  - Matching = 1-valid matching

Resulting Class of Problems

- Maximum Weighted K-Valid Matching problem (MWKVMP)

$$\text{maximize} \sum_{e \in M} w_e$$

$$\text{subject to } M \in S_K$$

$$\text{where } S_K \text{ is the set of all K-Valid matchings of G}$$

- Unweighted Version: Maximum K-Valid Matching Problem (MKVMP)

What value of K is optimal?

- The optimal value of K depends on many factors
  - Node density
  - Physical layer
    - For IEEE 802.11 networks (DSSS PHY), K = 2 seems to be the best choice
    - For EDGE networks, K=3 performs better than K=2 for a wide range of node densities
    - The ability to perform rate control also effects optimal K

Hardness Results

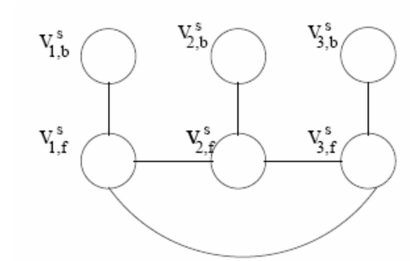
- MKVMP (decision version) is NP-Hard for K >1
  - Main Idea: Reduction from 3-CNF-SAT to MKVMP
- CNF – conjunctive normal form
  - A formula is in 3-CNF if it is AND of one or more clauses, each of which is an OR of exactly three distinct literals
- Example:

Clause 1

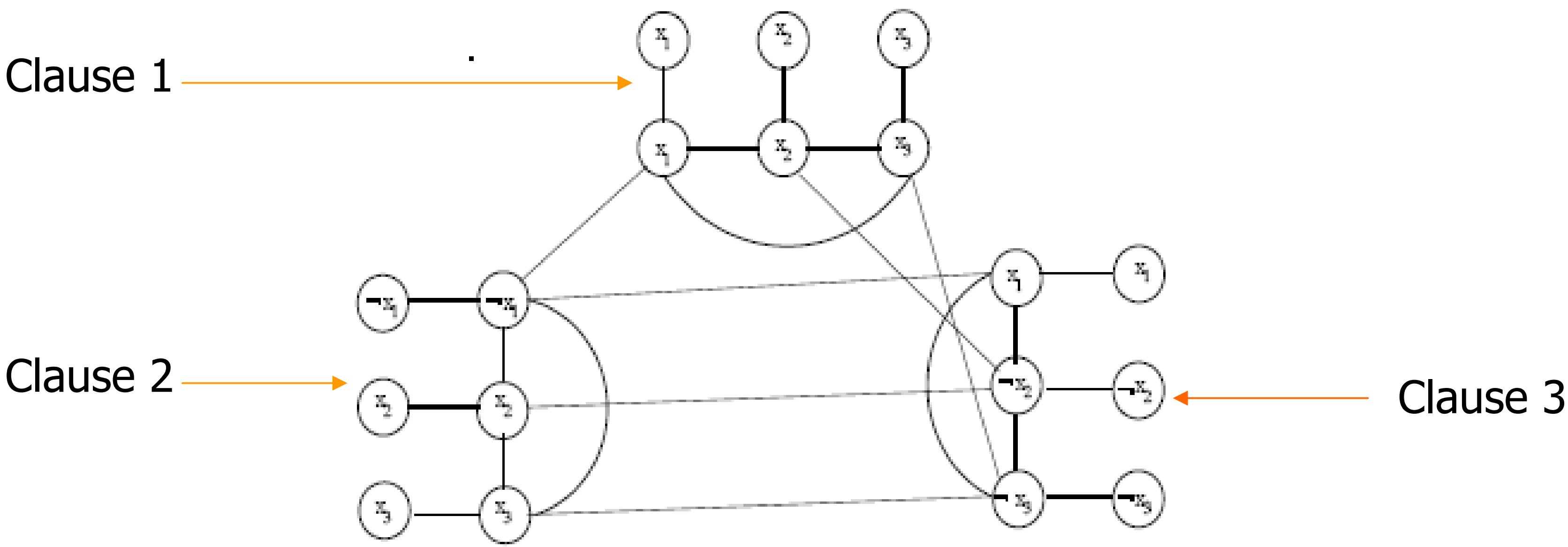
$$\Phi = (x_1 \vee x_2 \vee x_3) \wedge (\neg x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \neg x_2 \vee \neg x_3)$$

Hardness Proof

- Consider K=2
- Gadget used for a clause



$$\Phi = (x_1 \vee x_2 \vee x_3) \wedge (\neg x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \neg x_2 \vee \neg x_3)$$



Approximability Results

- Let η be such that $(|V| + K\,|E|)^{\eta} = \Theta(V)$.
- MKVMP for K > 1 is not approximable within
  - $|V|^{\eta/2-\varepsilon}$ for any $\varepsilon > 0$, unless NP = P
  - $|V|^{\eta-\varepsilon}$ for any $\varepsilon > 0$, unless NP = ZPP
- Note: Sparse graphs are not good!
- Proof Technique: Reduction from Maximum Independent Set Problem (MISP) to MKVMP

Approximability Results (Contd.)

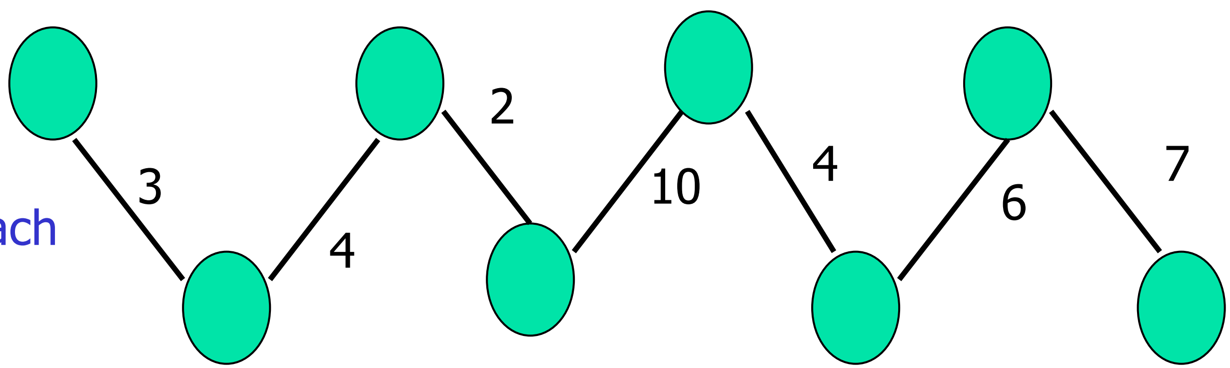
- MWKVMP is approximable within $\Theta\left(\frac{|E|}{(\log E)^2}\right)$
- Proof Technique: Reduction from MWKVMP to Vertex Weighted Maximum Independent Set problem (VWMISP)

Greedy Approach

Greedy Weighted K-Valid Matching Algorithm$(G = (V, E), w : E \rightarrow \mathbb{R}, M)$

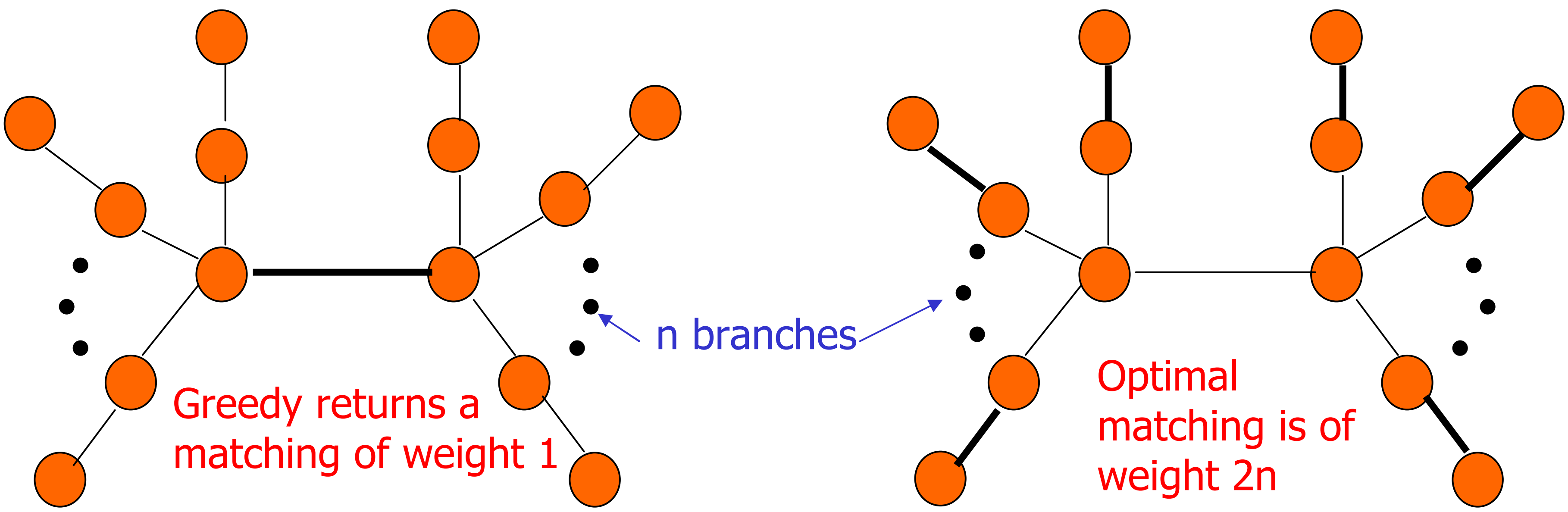
1. $M := \phi$ and $i := 1$.
2. Arrange edges of E in descending order of weight, starting with $e_1, e_2, \ldots$.
3. If $M \cup e_i$ is a valid K-valid matching, then $M := M \cup e_i$. $i := i + 1$.
4. Repeat Step 3 for all edges in E.

Greedy Approach
for K=2



Performance of Greedy

- K=1 → 2-approximation algorithm
- K>1 → Performance can be arbitrarily bad!



All edges have unit weight and K = 2

Performance of Greedy

- Some Terminology
  - *K-hop Interference set* $I_K(e)$ of an edge e is the set of edges within K hops of e
  - A subset S of $I_K(e)$ is *K-maximal* if no edge e_1 belonging to $I_K(e)$ can be added to it, while ensuring that $S \cup e$ is a K-valid matching
  - K-hop interference degree $d_K(e)$ of an edge e is defined as follows:

$$d_K(e) = \max_{S \subseteq I_K(e) : S \text{ is } K-\text{maximal}} |S|$$

Performance of Greedy (Contd.)

- More Terminology
  - K-hop interference degree of a graph G=(V,E) is defined as follows:

$$d_K(G) = \max_{e \in E} d_K(e)$$

- For a graph G, the greedy approach returns a matching whose weight is within a factor of $d_K(G)$ of the optimal matching
- Bad News: $d_K(G)$ can be of the order of |E|

MWKVM for Specific Graphs

- Geometric graphs
  - Vertices placed on the plane
  - Two vertices are connected if and only if they are within distance r of each other
- Why look at geometric graphs?
  - Same power level + Same noise level → underlying connectivity graph of a wireless network is indeed geometric!
- Results can easily be extended to disk graphs or (r,s)-civilized graphs

Results for Geometric Graphs

- Greedy approach works quite well for geometric graphs
  - $d_K(G) \leq 49$ for all K and all geometric graphs G!
- Polynomial time approximation scheme (PTAS) for MWKVMP
  - Returns a matching of weight within $(1+\epsilon)$ of an optimal matching for any arbitrary $\epsilon > 0$, in polynomial time

Implications

- For wireless networks whose connectivity graph is geometric (using our results and those in [Lin and Shroff, INFOCOM 2005]):
  - Greedy approach (respectively, PTAS) can be used to construct a scheduling policy that guarantees a throughput within a factor of 49 (respectively, $1+\epsilon$) of the optimal, under a K-hop interference model for any arbitrary K

Related Works

- Node-exclusive interference model (K=1)
  - Synchronous congestion control, performance of MM algorithm [Lin and Shroff, INFOCOM 2005]
  - Asynchronous congestion control, performance of regulated MM algorithm [Wu and Srikant, CDC 2005] & [Bui et al., INFOCOM 2006]
- IEEE 802.11 type interference model (K=2)
  - Performance of "maximal scheduling policy" [Wu and Srikant, INFOCOM 2006]
  - Approximability results for MKVMP; PTAS and distributed approximation algorithm for MKVMP [Balakrishnan et al., IEEE JSAC 2004]
- General interference model (contention matrix based)
  - Performance of maximal scheduling policy [Chaporkar et al., Tech Report, UPenn, 2005]

Work in Progress

- Performance of maximal scheduling policy in case of geometric graphs
  - Achieves a throughput within a factor of 49 of the optimal provided no rate control is allowed and all traffic is single-hop (MAC layer)
  - Currently working on extending this result to a setting with multi-hop traffic and rate control

- Fully distributed algorithms for maximal scheduling policy
  - K=2 case is done
  - K>2 case is currently being studied

Thanks for Listening!

Questions?