## Strong Edge Coloring for Channel Assignment in Wireless Radio Networks

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## Wireless broadcast

A node $u$ can transmit to every node within its broadcast range Pairs of nodes talk on a mutually decided frequency channel


Communication is bidirectional since both data and acknowledgment packets need to be sent

## Channel assignment as a graph problem

Channel assignment or Link scheduling problem:
Assign a frequency channel to each pair of transceivers that want to communicate

Model the wireless network as a graph $G=(V, E)$ :
Nodes in $V$ are the $n$ transceivers
Edges in $E$ are pairs of transceivers that can communicate directly

Channel assignment or Link scheduling problem: Assign a frequency channel to each edge of $E$

## Primary interference



Primary interference at $v$ from two nodes, both transmitting to $v$, using the same channel simultaneously

## Secondary interference



Secondary interference at $v$ from two simultaneous transmissions
Tranmissions $u \rightarrow v$ and $w \rightarrow z$ interfere at $v$
Transmissions are intended for two different receivers

## Edge coloring

Given a graph $G=(V, E)$ representing the wireless network

An edge coloring with $k$ colors is a function

$$
f: E \rightarrow\{1,2,3, \ldots, k\}
$$

that assigns an integer (i.e., a color) to each edge of $G$

The edge coloring $f$ assigns one of $k$ available channels to every pair of communicating transceivers

A partial edge coloring colors a subset of the edges

## Strong edge coloring

Coloring constrained to avoid interference Two edges are within distance-2 of each other if they are adjacent or if they are both incident on a common edge


## Strong edge coloring

Coloring constrained to avoid interference
Two edges are within distance-2 of each other if they are adjacent or if they are both incident on a common edge


A strong edge coloring is a coloring that assigns distinct colors to every pair of edges within distance-2 of each other

A strong edge coloring is a conflict-free channel assignment avoiding primary and secondary interference

## Channel assignment in wireless networking



Solid edges must get different colors because of dashed edges
S. Ramanathan. A Unified Framework and Algorithm for Channel Assignment in Wireless Networks. Wireless Networks 5 (1999), pp. 81-94.

## Strong edge coloring in graph theory

Number of colors $\chi$ needed is $\Delta \leq \chi \leq 2 \Delta^{2}-2 \Delta+1$
NP-hard to determine if a graph is strong edge colorable with $k$ colors for $k \geq 4$ [Mahdian 2000]

NP-hard to determine if a bipartite, 2-inductive graph is strong edge colorable with 5 colors [Erickson,Thite,Bunde 2002]

Fixed parameter tractable: A strong edge coloring of $G$ with the fewest colors can be computed in polynomial time if the treewidth of $G$ is bounded [Salavatipour 2004]

Erdős-Nes̆etřil conjecture (1985): strong edge-coloring number of a graph is at most $\frac{5}{4} \Delta^{2}$ when $\Delta$ is even and $\frac{1}{4}\left(5 \Delta^{2}-\right.$ $2 \Delta+1)$ when $\Delta$ is odd.

## Algorithms for strong edge coloring

We study and analyze sequential and distributed strong edge coloring algorithms for various graph classes

Different graph classes model different aspects of wireless networks; for example:
disk graphs model geographic locality of communication $c$-inductive graphs model sparsity

Basic outline of our algorithms:
Order or partially order the edges
Color edges greedily, choosing first available color

Let Opt be the minimum number of colors used in a strong edge coloring of $G$; we use at most $\mathbf{c}$ - Opt colors, where $c$ is a constant

## D2EC( $G$ ): Strong edge coloring $G$ with the fewest colors

Assign a color to each (bidirected) edge
Each color is a specific frequency from the available spectrum Strong edge coloring: Any two edges within distance 2 of each other must get different colors


We minimize the number of colors in a strong edge coloring of the whole graph.

## Unit-disk graphs

Choose nodes in coordinate order Greedily color incident edges with first available color


Our algorithm uses at most 8 OPT +1 colors

## Unit-disk graphs



An edge $(w, z)$ within distance-2 of $(u, v)$ must have one endpoint in $D_{u} \cup D_{v}$

If $(w, z)$ is colored before $(u, v)$, both $w$ and $z$ must precede $u$

## Lower bound for unit-disk graphs

If color $k$ is used by the greedy algorithm for edge ( $u, v$ ), then $k-1$ colors must appear on neighboring edges


Neighboring edges are incident on $\leq 8$ cliques

Two edges incident on a clique

Our algorithm uses at most 8 Opt +1 colors

## $c$-inductive graphs

## Definition:

A graph with at most $c$ vertices is $c$-inductive
A graph $G$ is $c$-inductive if it has a vertex $v$ of degree $\leq c$ such that $G-v$ is $c$-inductive
Vertices can be sorted in inductive order


Every graph $G$ is $c$-inductive for some $c$ in the range $\delta(G) \leq c \leq \Delta(G)$

A tree is 1 -inductive; a planar graph is 5 -inductive
Graphs of bounded genus, bounded treewidth, $d$-plyneighborhood systems are $c$-inductive for small $c$

## Coloring $c$-inductive graphs



Compute an inductive ordering of the nodes of $G$

For the $i$ th node $u_{i}$, greedily color all previously uncolored edges incident on $u_{i}$-each uncolored edge is given the first available color

Our algorithm uses at most $4 c \Delta$ colors; since $\Delta$ colors are necessary, this is an $O(c)$ approximation

## $(r, s)$-civilized graphs

A graph that can be drawn in $E^{d}$ such that the length of each edge is $\leq r$ and the distance between any two points is $\geq s$


Can model geographic networks with occlusions due to topography or other obtstacles

We have a 2-approximation algorithm, using Mahdian's exact algorithm for bounded-treewidth graphs

Running time (only) is a function of $r / s$

## D2EC $(G, k)$ : Coloring most edges of $G$ given $k$ colors

Let $G=(V, E)$ be a disk graph modeling a wireless network Given a palette of $k$ colors, $k$ fixed

Maximize the number of edges colored in a partial strong edge coloring of $G$ with at most $k$ colors

Equivalently, maximize the number of pairs of transceivers that can communicate without interference

We maximize the number of edges of $G$ colored in a partial strong edge coloring with the given $k$ colors

Let Opt be the maximum number of edges colored in a partial strong edge coloring of $G$ with $k$ colors

We color at leat c $\cdot$ Opt edges, where $c$ is a constant

## Distributed model: synchronized broadcast model

Computation proceeds in rounds

In each round, a node is either active or inactive

In each round, only active nodes transmit; inactive nodes remain silent

Every transmission is a broadcast

Node $v$ receives a message from node $w$ if and only if:
$v$ is silent and $w$ transmits
no collision: $w$ is the only neighbor of $v$ to transmit

## Assumptions / Caveats

Each node is aware of the number of its neighbors but not necessarily their IDs

Each node knows its active degree, i.e., the number of its neighbors that are active in the current round

At the beginning of each round, the active degree of every (active) vertex is computed in a distributed fashion, in $\rho$ rounds

Every node can detect if a collision happens, i.e., node $v$ can detect if multiple neighbors of $v$ transmit in the same round

## Distributed algorithm for unit-disk graphs

Randomized distributed algorithm for $\operatorname{D} 2 \mathrm{EC}(G, k)$ where $G$ is a unit-disk graph

In $O(k \rho \log n)$ rounds, the algorithm colors at least $c$-Opt edges, with high probability

A simplified variant of Luby's (1986) randomized algorithm for max independent set

In each round, each vertex executes a simple protocol: compute $\rightarrow$ send $\rightarrow$ receive

We use and refine results from an earlier paper:
H. Balakrishnan, C. L. Barrett, V. S. Anil Kumar, M. V. Marathe, S. Thite. The distance-2 matching problem and its relationship to the MAC-layer capacity of ad-hoc wireless networks. IEEE J. Selected Areas in Communication, 22(6):1069-1079, 2004.

## In each round:

Each active vertex decides to wake up by flipping a coin

Nodes that are awake broadcast to all their neighbors

If a node (active or not) hears another node, it broadcasts this knowledge

If an active node does not hear another node, then it belongs to a subset $S$

If a node hears one or more nodes, then it belongs to another subset $T$

## Analysis

Wake-up probability of node $v$ is inversely proportional to its active degree; since coin flips are independent, the probability of a collision is small

After $O(\rho \log n)$ rounds with high probability, each node is in either $S$ or $T$
$S$ is a strong independent set; in another $O(1)$ rounds, nodes in $S$ compute a strong matching $M \subseteq E$
$|M|$ is guaranteed to be 'large', i.e., at least a constant times a maximum matching

For every vertex $u$, at most $O(1)$ edges within distance- 2 can be in a strong matching
For every vertex $u$, at least one edge within distance- 2 is chosen by our algorithm
All edges in $M$ are greedily assigned a color
Repeat for $G-M$
Result: $O(1)$-approximate maximum partial strong edge coloring, in $O(k \rho \log n)$ rounds with high probability.

## Summary

We formulate interference-free channel assignment in wireless networks as the strong edge coloring problem on graphs

We study various graph classes modeling different aspects of realistic wireless networks

We give sequential approximation algorithms to minimize the number of channels in a channel assignment

We give a distributed protocol to maximize the number of transceivers that can successfully communicate when we are given a fixed frequency spectrum

## Grazie!

## Experiments

Experiments: Greedy distributed algorithm for D2EC( $G$ ) on random disk graphs with power-law radii distribution
$n$ random points in a unit square; broadcast radius follows power-law distribution-number of nodes with radius $r$ proportional to $r^{\alpha}$


Number of points (log scale)



Number of colors used to color all edges increases with $n$, but slower than theoretical bound for general graphs

## Experiments: Greedy distributed algorithm for D2EC $(G, k)$ on

 random unit-disk graphs$n$ random points in a unit square; each with the same broadcast radius $r$ chosen randomly

Fraction of edges colored


A large fraction of edges are colored in just 1 round, with a palette size of $\Delta^{2}$

Experiments: Greedy distributed algorithm on random unit-disk graphs


Number of edges colored VS.
Number of colors

D2EC(k), $n=10000$, unit square


